EX FEBRERO 2024 S2 - CÁLCULO GRADO ECONOMÍA UNED

PROBLEMA 1

Hallar el polinomio de Taylor de grado 2 en un entorno de x=1 para la función f(x) = xln(x)

$$P_2(x) = f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2$$

$$f(1) = 1 \cdot \ln 1 = 1 \cdot 0 = 0$$

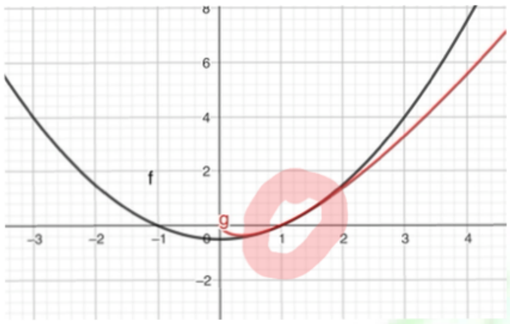
$$f'(x) = 1 \cdot \ln(x) + x \cdot \frac{1}{x} = \ln x + 1 \Rightarrow f'(1) = \ln 1 + 1 = 1$$

$$f''(x) = \frac{1}{x} \Rightarrow f''(1) = \frac{1}{1} = 1$$

$$P_2(x) = 0 + \frac{1}{1}(x-1) + \frac{1}{2}(x-1)^2$$

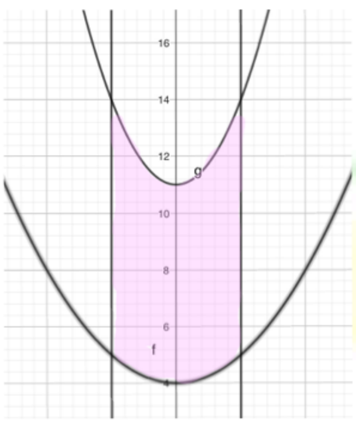
$$= x - 1 + \frac{1}{2}(x^2 - 2x + 1) = \cancel{x} - 1 + \frac{1}{2}x^2 - \cancel{x} + \frac{1}{2}$$

$$\boxed{P_2(x) = \frac{1}{2}x^2 - \frac{1}{2}}$$



PROBLEMA 2

Calcula el área de la región comprendida entre la curvas f(x)= $x^2 + 4$; g(x) = $3x^2 + 11$. Y las rectas x=-1 y x=1



$x=-1$ $\quad$ $x=1$

1) $f(x) = g(x)$

$$x^2 + 4 = 3x^2 + 11$$

$$-7 = 2x^2 \quad \nexists$$

2) $$\left| \int_{-1}^{1} 3x^2 + 11 - (x^2 + 4)\, dx \right| = A$$

$$\int_{-1}^{1} (3x^2 + 11 - x^2 - 4)\, dx = \int_{-1}^{1} 2x^2 + 7\, dx =$$

Rec: $\int x^n dx = \frac{x^{n+1}}{n+1}$

$$= 2\frac{x^3}{3} + 7x \Big]_{-1}^{1} = \left(\frac{2}{3} \cdot 1^3 + 7 \cdot 1\right) - \left(2 \cdot \frac{(-1)^3}{3} + 7 \cdot (-1)\right)$$

$$= \frac{2}{3} + 7 + \frac{2}{3} + 7 = 14 + \frac{4}{3} = \frac{46}{3}\, u^2$$

PROBLEMA 3

Halla la diferencial de la siguiente función en (1, -1, 1) : $f(x, y, z) = (3x^2yz + 3xy^2z + 3xyz^2)$

$$df(x,y,z) = \frac{\partial f}{\partial x}(x,y,z)\,dx + \frac{\partial f}{\partial y}(x,y,z)\,dy + \frac{\partial f}{\partial z}(x,y,z)\,dz$$

$$= f'_x(x,y,z)\,dx + f'_y(x,y,z)\,dy + f'_z(x,y,z)\,dz$$

$$\frac{\partial f}{\partial x}(x,y,z) \underset{\substack{\downarrow \\ y,z \text{ const}}}{=} 6xyz + 3y^2z + 3yz^2$$

$$\frac{\partial f}{\partial y}(x,y,z) \underset{\substack{\downarrow \\ x,z \text{ const}}}{=} 3x^2z + 6xyz + 3xz^2$$

$$\frac{\partial f}{\partial z}(x,y,z) \underset{\substack{\downarrow \\ x,y \text{ const}}}{=} 3x^2y + 3xy^2 + 6xyz$$

$$df(x,y,z) = (6xyz + 3y^2z + 3yz^2)\,dx + (3x^2z + 6xyz + 3xz^2)\,dy + (3x^2y + 3xy^2 + 6xyz)\,dz$$

$$df(1,-1,1) = (-6 + 3 + (-3))\,dx + (\overbrace{3 + (-6) + 3}^{=0})\,dy + (-3 + 3 + (-6))\,dz$$

$$= -6\,dx - 6\,dz$$

PROBLEMA 4

Enunciar el criterio d'Alembert o del cociente.

$\sum a_n$

$\{a_n\}$ serie de términos positivos

$\lim_{n\to\infty} a_n = 0$

Si existe $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = R \Rightarrow \begin{cases} R < 1 & \sum a_n \text{ convergente} \\ R > 1 & \sum a_n \text{ diverge} \\ R = 1 & \text{este criterio no decide.} \end{cases}$

$$\sum_{n=1}^{\infty} \frac{5^n}{n!} \qquad \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{5^{n+1}}{(n+1)!}}{\frac{5^n}{n!}} = \lim_{n\to\infty} \frac{5^{n+1} \cdot n!}{5^n \cdot (n+1)!}$$

Recordad: $A^{n+m} = A^n \cdot A^m \qquad 5^{n+1} = 5^n \cdot 5$

$$(n+1)! = (n+1) \cdot \underbrace{n \cdot (n-1) \cdot \ldots \cdot 1}_{n!}$$

$$= \lim_{n\to\infty} \frac{5^n \cdot 5 \cdot n!}{5^n \cdot (n+1) \cdot n!} =$$

$$= \lim_{n\to\infty} \frac{5}{n+1} \longrightarrow 0$$

Como $R = 0$ $(R<1) \Rightarrow$

$\Rightarrow \sum a_n$ converge.

PROBLEMA 5

a) Halla el dominio y estudia la existencia de asíntotas.
b) Determina los intervalos de crecimiento y decrecimiento.
c) Determina los extremos relativos.

$$f(x) = \frac{x^2 + 2x}{e^x} \qquad e^x \neq 0$$

a) Dominio: $\operatorname{dom} f = \mathbb{R}$

Asíntotas: A. verticales: No Hay

A. Horizontales:

L'Hop — Hay A. Horiz $y=0$ ($x \to +\infty$)

$$\lim_{x \to +\infty} \frac{x^2+2x}{e^x} = \frac{\infty}{\infty} \text{ Ind} \Rightarrow \lim_{x \to \infty} \frac{2x+2}{e^x} = \frac{\infty}{\infty} \Rightarrow \lim_{x \to \infty} \frac{2}{e^x} = \frac{2}{\infty} = 0$$

$$\lim_{x \to -\infty} \frac{x^2+2x}{e^x} = \lim_{x \to -\infty} e^{-x} \cdot (x^2+2x) = e^{\infty} \cdot (\infty) = \infty$$

A. oblicuas: No hay $x \to +\infty$

b) Monotonía

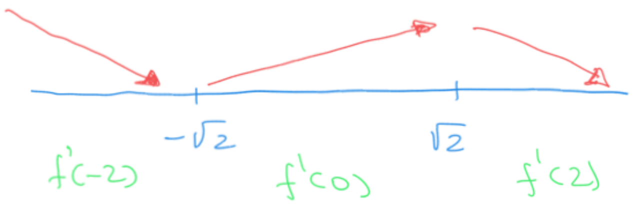
$$f'(x) = \frac{(2x+2) \cdot e^x - (x^2+2x) \cdot e^x}{(e^x)^2} = \frac{(\cancel{2x}+2) - (x^2+\cancel{2x})}{e^x} = \frac{-x^2+2}{e^x} = 0$$

e^x (+)

$f'(x) > 0$ crece

$f'(x) < 0$ decrece

$$\Rightarrow -x^2+2=0 \Rightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2} \text{ posibles extremos}$$



$$f'(-2) = \frac{-(-2)^2+2}{+} < 0$$

$$f'(0) = \frac{-0^2+2}{+} = 2 > 0$$

$$f'(2) = \frac{-2^2+2}{e^2} < 0$$

Crece: $(-\sqrt{2}, \sqrt{2})$

decrece: $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, +\infty)$

$$f(x) = \frac{x^2+2x}{e^x}$$

c) Extremos relativos:

$x = -\sqrt{2}$ mínimo $\Rightarrow f(-\sqrt{2}) = \frac{(-\sqrt{2})^2 + 2(-\sqrt{2})}{e^{-\sqrt{2}}} = \frac{2-2\sqrt{2}}{e^{-\sqrt{2}}}$

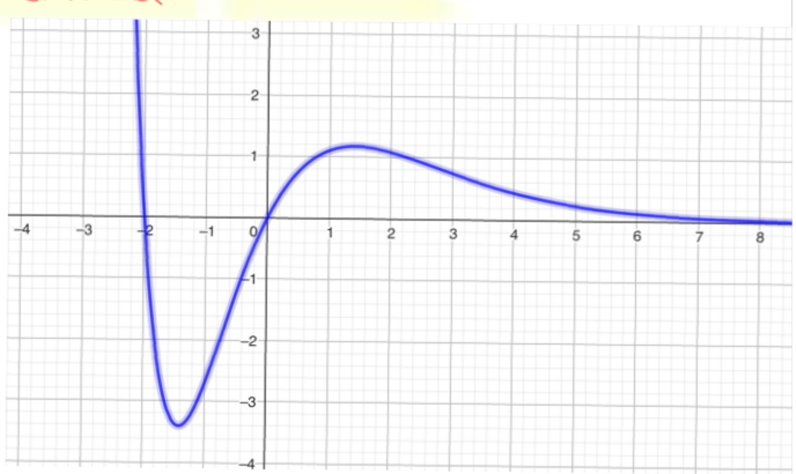
$x = \sqrt{2}$ máximo $\Rightarrow f(\sqrt{2}) = \frac{\sqrt{2}^2 + 2\sqrt{2}}{e^{\sqrt{2}}} = \frac{2+2\sqrt{2}}{e^{\sqrt{2}}}$

mínimo $\left(-\sqrt{2}, \frac{2-2\sqrt{2}}{e^{-\sqrt{2}}}\right)$ máximo $\left(\sqrt{2}, \frac{2+2\sqrt{2}}{e^{\sqrt{2}}}\right)$

Otra manera $\Rightarrow$ Hacer la segunda derivada

$f''(-\sqrt{2}) > 0 \Rightarrow$ mínimo

$f''(\sqrt{2}) < 0 \Rightarrow$ máximo



PROBLEMA 6

Una compañía tiene la siguiente función de costes para los tres bienes, (x, y, z), que produce:
$f(x, y, z) = 8x^2 + 2y^2 + 2z^2 + 4xy - 40x - 16y - 20z$. Determina la producción que hace mínimos los costes.

$\frac{\partial f}{\partial x} = f'_x = 16x + 4y - 40 = 0$ (y, z const) $\xrightarrow{:4}$ $4x + y - 10 = 0$

$\frac{\partial f}{\partial y} = f'_y = 4y + 4x - 16 = 0$ (x, z const) $\xrightarrow{:4}$ $-\ x + y - 4 = 0$ $\rightarrow$ $2 + y - 4 = 0$

$3x - 6 = 0 \Rightarrow \boxed{x = 2}$ $\quad \boxed{y = 2}$

$\frac{\partial f}{\partial z} = f'_z = 4z - 20 = 0$ (x, y const) $\rightarrow \boxed{z = 5}$

Posible mínimo $(2, 2, 5)$

Matriz Hessiana:

$\frac{\partial f}{\partial x} = f'_x = 16x + 4y - 40 = 0$ → $\frac{\partial^2 f}{\partial x^2} = 16$ (f''_{xx}), $\frac{\partial^2 f}{\partial xy} = 4$, $\frac{\partial^2 f}{\partial xz} = 0$

$\frac{\partial f}{\partial y} = 4y + 4x - 16$ → $\frac{\partial^2 f}{\partial yx} = 4$ (f''_{yx}), $\frac{\partial^2 f}{\partial y^2} = 4$ (f''_{yy}), $\frac{\partial^2 f}{\partial yz} = 0$ (f''_{yz})

$\frac{\partial f}{\partial z} = 4z - 20$ → $\frac{\partial^2 f}{\partial zx} = 0$, $\frac{\partial^2 f}{\partial zy} = 0$, $\frac{\partial^2 f}{\partial z^2} = 4$

$$Hf(x,y,z) = \begin{pmatrix} 16 & 4 & 0 \\ 4 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

H_1, H_2

$\det H = 16 \cdot 4 \cdot 4 - 4 \cdot 4 \cdot 4 > 0 \Rightarrow$ es un extremo

$\det H_2 = 16 \cdot 4 - 4 \cdot 4 > 0$

$H_1 = 16 > 0$

definida positiva $\Downarrow$ mínimo

el val $(2, 2, 5)$ es un mínimo.

el valor mínimo es $\underline{f(2,2,5)}$