EJERCICIOS INTEGRAL INDEFINIDA

1. Calcular: $\int \frac{e^x}{(1-3e^x)^2}\,dx =$

$$= \frac{1}{-3}\int -3e^x(1-3e^x)^{-2}\,dx = -\frac{1}{3}\,\frac{(1-3e^x)^{-2+1}}{-2+1} + C_1 =$$

$$= +\frac{1}{3}\,\frac{(1-3e^x)^{-1}}{+1} + C_1 =$$

$$= \boxed{\frac{1}{3(1-3e^x)} + C_1}$$

2. Resolver: $\int x \ln x^2\,dx = \int x\, 2\ln x\, dx = 2\int x \cdot \ln x\, dx =$ *

ALPES

$$= \left\{ \begin{array}{ll} u = \ln x & du = \frac{1}{x}\,dx \\ v = \frac{x^2}{2} & dv = x\,dx \end{array} \right\} =$$

$$\int x \ln x\, dx = \frac{x^2}{2}\ln x - \int \frac{x^{\cancel{2}}}{2}\,\frac{1}{\cancel{x}}\,dx = \frac{x^2}{2}\ln x - \frac{1}{2}\int x\,dx =$$

$$= \frac{x^2}{2}\ln x - \frac{1}{2}\,\frac{x^2}{2} + C_1 = \frac{x^2}{2}\ln x - \frac{x^2}{4} + C_1$$

$$* = 2\left(\frac{x^2}{2}\ln x - \frac{x^2}{4} + C_1\right) = x^2 \ln x - \frac{x^2}{2} + \underbrace{2C_1}_{K} =$$

$$= \boxed{x^2 \ln x - \frac{x^2}{2} + K}$$

3. Calcular $\int x^2 e^{3x}dx = \left\{\begin{matrix} u = x^2 & du = 2x\,dx \\ v = \frac{1}{3}e^{3x} & dv = e^{3x}dx \end{matrix}\right\} =$

ALPES

$v = \frac{1}{3}\int 3\,e^{3x}dx$

$$= \frac{1}{3}x^2e^{3x} - \int \frac{1}{3}e^{3x}\cdot 2x\,dx =$$

$$= \frac{1}{3}x^2e^{3x} - \frac{2}{3}\int x\cdot e^{3x}dx = \left\{\begin{matrix} u = x & du = dx \\ v = \frac{1}{3}e^{3x} & dv = e^{3x}dx \end{matrix}\right\} =$$

$$= \frac{1}{3}x^2e^{3x} - \frac{2}{3}\left[\frac{1}{3}xe^{3x} - \int \frac{1}{3}e^{3x}dx\right] =$$

$$= \frac{1}{3}x^2e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{9}\underbrace{\int e^{3x}dx}_{\frac{1}{3}e^{3x}} =$$

$$= \frac{1}{3}x^2e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{9}\,\frac{1}{3}e^{3x} + C$$

$$= \boxed{e^{3x}\left(\frac{1}{3}x^2 - \frac{2}{9}x + \frac{2}{27}\right) + C}$$

4. Resolver: $\int lnx\, dx$ $= \left\{ \begin{array}{ll} u = \ln x & du = \frac{1}{x} dx \\ v = x & dv = dx \end{array} \right\} =$

$$= x \ln x - \int x \frac{1}{x} dx = x \ln x - \int dx =$$

$$= x \ln x - x + C =$$

$$= \boxed{x(\ln x - 1) + C}$$

ADEFACIL

5. Examen Enero 2020 1ª Semana

2. El beneficio marginal de una empresa, en función de la cantidad x invertida en publicidad, medida en euros, viene dada por la función $B'(x) = 100e^{-\frac{x}{5}} - 20xe^{-\frac{x}{5}} - 1$

   Si dicho beneficio está también expresado en euros, hállese $B(x)$ sabiendo que el beneficio es de 2.125.000 euroscuando se gastran 10.000 euros en publicidad. ¿Cúal es es el beneficio si la inversión en publicidad es de 20.000 euros?.
   Tómese $e^{-2} = 0{,}135$ y $e^{-4} = 0{,}018$

$$B(x) = \int B'(x)\,dx = \int \left(100e^{-x/5} - 20xe^{-x/5} - 1\right)dx =$$

$$= 100\underbrace{\int e^{-x/5}\,dx}_{①} - 20\underbrace{\int x e^{-x/5}\,dx}_{②} - \int dx = *_1$$

$$① \quad (-5)\int -\tfrac{1}{5}e^{-\frac{x}{5}}\,dx = -5\cdot e^{-x/5} + C_1$$

$$② \quad \int x e^{-x/5}\,dx = \left\{\begin{array}{ll} u = x & du = dx \\ v = -5\cdot e^{-x/5} & dv = e^{-x/5}\,dx \end{array}\right\} =$$

ALPES

$$= -5xe^{-x/5} - \int -5e^{-x/5}\,dx =$$

$$= -5xe^{-x/5} + 5\underbrace{\int e^{-x/5}\,dx}_{-5e^{-x/5}} + C_2 =$$

$$= -5xe^{-x/5} - 25e^{-x/5} + C_2$$

ADEFACIL

$*_1$

$$= 100\left(-5 \cdot e^{-x/5}\right) - 20\left(-5x e^{-x/5} - 25 e^{-x/5}\right) - x + C_1 =$$

$$= -\cancel{500 e^{-x/5}} + 100x\, e^{-x/5} + \cancel{500 e^{-x/5}} - x + C_1 =$$

$$= 100x\, e^{-x/5} - x + C_1$$

$$\boxed{B(x) = 100x\, e^{-x/5} - x + C_1}$$

$$B(x=10.000) = 100 \cdot 10.000 \cdot e^{-10.000/5} - 10.000 + C_1 =$$

$$= 2.125.000$$

$$1.000.000\, e^{-2000} - 10.000 + C_1 = 2.125.000$$

$$1000.000\left(e^{-2}\right)^{1000} + C_1 = \underbrace{2.125.000 + 10.000}_{2.135.000}$$

$$1.000.000\left(0'135\right)^{1000} + C_1 = 2.135.000$$

$$C_1 = 2.135.000 - 1000.000\left(\frac{135}{1000}\right)^{1000}$$

$$C_1 = 2.135.000 - 1000.000\left(0'135\right)^{1000}$$

$$\boxed{B(x) = 100x\, e^{-x/5} - x + 2.135.000 - 1000.000\left(0'135\right)^{1000}}$$

$$B(x) = x\left(100\, e^{-x/5} - 1\right) + c_1$$

$$B(20.000) = 20.000\left(100\, e^{-20.000/5} - 1\right) + c_1 =$$

$$= 20.000\left(100\, e^{-4000} - 1\right) + c_1 =$$

$$= 20.000\left(100\left(e^{-4}\right)^{1000} - 1\right) + c_1 =$$

$$= \boxed{20.000\left(100\left(0'018\right)^{1000} - 1\right) + 2.135.000 - 1.000.000\left(0'135\right)^{1000}}$$

6. Examen Enero 2020 2ª Semana

2. Si x representa el nivel de producción de una empresa, el coste marginal es de $C'(x) = 5.000(x+20)^{-2} * Ln(x+20)$ euros por unidad, y los costes fijos, de 2.000 euros. Determinar $C(x)$ y el coste de producir 80 unidades.
Tómese $Ln(20) = 3$ y $Ln(100) = 4{,}6$

$$C(x) = \int C'(x)\,dx = \int 5000(x+20)^{-2}\cdot \ln(x+20)\,dx =$$

$$= 5000\int (x+20)^{-2}\cdot \ln(x+20)\,dx = \left\{\begin{array}{l} t = x+20 \\ dt = dx \end{array}\right\} =$$

ALPES

$$= 5000\int t^{-2}\cdot \ln(t)\,dt = \left\{\begin{array}{ll} u = \ln t & du = \frac{1}{t}dt \\ v = \frac{t^{-1}}{-1} = -\frac{1}{t} & dv = t^{-2}dt \end{array}\right\} =$$

$$= 5000\left[-\frac{1}{t}\ln t - \int -\frac{1}{t}\frac{1}{t}\,dt\right] =$$

$$= 5000\left[-\frac{1}{t}\ln t + \int \frac{1}{t^2}\,dt\right] =$$

$$= 5000\left[-\frac{1}{t}\ln t - \frac{1}{t} + C\right] =$$

$$= -\frac{5000}{t}\ln t - \frac{5000}{t} + K =$$

$$= -\frac{5000}{t}(\ln t - 1) + K$$

$$C(x) = 5000 \int (x+20)^{-2} \cdot \ln(x+20)\, dx = \left\{ t = x+20 \right\} =$$

$$= \boxed{-\frac{5000}{x+20}\left(\ln(x+20) - 1\right) + K}$$

$$C(0) = 2.000$$

$$C(0) = \frac{-5000}{20}\underbrace{\left(\underbrace{\ln(20)}_{3} - 1\right)}_{2} + K = 2.000$$

$$-\frac{5000}{20} \cdot 2 + K = 2000$$

$$-500 + K = 2000 \longrightarrow K = 2000 + 500 =$$
$$= 2500$$

$$\boxed{C(x) = \frac{-5000}{x+20}\left[\ln(x+20) - 1\right] + 2500}$$

$$C(80) = \frac{-5000}{80+20}\left[\ln(80+20) - 1\right] + 2500 =$$

$$= -50\left[\ln(100) - 1\right] + 2500 =$$

$$= -50\left(4'6 - 1\right) + 2500 =$$

$$= -\underbrace{50 \cdot 3'6}_{-180} + 2500 = \boxed{2350}$$